Algorithms for Generating a Skew-T, Log P Diagram and Computing Selected Meteorological Quantities

G. S. Stipanuk
Atmospheric Sciences Laboratory
White Sands Missile Range, New Mexico, 88002
October 1973
Report ECOM-5515

With Updates by Harold Reynolds
April 7, 1991

1. Abstract

This paper discusses numerical methods of computing meteorological quantities which are usually manually derived from analysis on a SKEW-T, log p diagram. The numerical methods used were selected for their simplicity and accuracy. A mathematical characterization of the SKEW-T and algorithm for computing several meteorological quantities are presented.

2. Symbols

CCL – Convective condensation level

C_P – Heat capacity of air at constant pressure

CT – Convective temperature

E – Actual vapor pressure

ES – Saturation vapor pressure

FR – Relative humidity

i, j, k – Indexes

L – Latent heat of vaporization of water

LCL – Lifting condensation level

M – Saturation vapor pressure over water

P – Pressure

P^* – Pressure correction

PB – Pressure at the bottom of a layer

PC – Pressure at the convective condensation level

PI – Pressure at the intersection

PM – Pressure at the top of the mixing layer

PS – Surface pressure

R – Gas Constant

T – Temperature

T^* – Temperature correction

TD – Dew point temperature

TDS – Dew point temperature at the surface

T_{DA} – Temperature on a dry adiabatic curve

TE – Pseudo equivalent temperature

TI – Temperature at an intersection

T_M – Temperature at the top of the mixing layer

T_{MR} – Temperature on a mixing ratio curve

T_{SA} – Temperature on a saturation adiabat curve

TW – Wet bulb temperature

W – Mixing ratio

$\overline{W}$ – Mean mixing ratio

X – Coordinate

Y – Coordinate

Z – Thickness of a layer

θ – Potential temperature

θ_e – Equivalent potential temperature

θ_S – Parameter for saturation adiabat

3. Introduction

The increasing availability of computing facilities, programmable calculators, and mini-computers allows many of the computations currently performed by manual graphics to be done by computer. This paper discusses numerical methods of computing meteorological quantities which are usually manually derived from analysis on a SKEW-T, log p diagram (or SKEW-T). The numerical methods used were selected for their simplicity and accuracy. A mathematical characterization of the SKEW-T and algorithms for computing several meteorological variables are presented. Finally, a discussion of the application of these methods and a FORTRAN program listing to accomplish the computations are included.

4. The SKEW-T, log p Diagram

The SKEW-T, log p diagram[i] is a thermodynamic chart with five families of curves, five types of scales and three data blocks. Various numerical information is also tabulated on the SKEW-T. This paper is concerned chiefly with the five families of curves which are associated with pressure, temperature, dry adiabat, saturation adiabat, and mixing ratio.

The first two families of curves, temperature and pressure, are used to locate points on the chart. An arbitrary coordinate system has been selected to measure distances. Let the origin correspond to the point at a temperature of 0°C (centigrade) and a pressure of 1000 mb (millibars). Take the X direction to be parallel to the pressure lines (horizontal), with positive X to the right. The point at a temperature of 1°C and a pressure of 1000 mb is on the positive side of the origin. The Y direction is perpendicular to the X direction. Positive Y is towards lower pressures (up). A point on the chart which is specified by its temperature and pressure may be transformed to X,Y coordinates by Equations (1) and (2). The components of the X,Y system are given in inches.[a]

$$X = 0.1408T - 10.53975 \log_{10} P + 31.61923 \tag{1}$$

$$Y = -11.5 \log_{10} P + 34.5 \tag{2}$$

The remaining three families of curves, TWR, TSA, and TDA, are given in Table 1. The temperatures are specified as a function of pressure and a parameter, the parameter serving as a means of specifying a particular curve of the family.

The temperature T at an arbitrary pressure on a saturation adiabat is determined by the bisection method.[b] The temperature is assumed to lie in the range -80°C to 40°C. An initial guess of -20°C is made and the correction T^* is computed. The correction term decreases by a factor of ½ after each correction. Terminating after 13 corrections gave satisfactory results. The algorithm for computing the temperature on a saturation adiabat is based on Equation 3:

$$\theta = \theta_e \exp\left(\frac{-LW}{C_P T}\right) \tag{3}$$

The latent heat of vaporization L and the heat capacity of air at constant pressure C_P are considered constant. Since it is not known how accurately the saturation adiabat could be determined from Eq. (3), Table 2 was constructed using List[ii] as a standard. The temperature on an arbitrary mixing ratio curve W is computed by first computing the saturation vapor pressure M. An approximation to the inverse saturation vapor pressure function is then used to compute the temperature.

In addition to the algorithms which generate the curves for each family, it is necessary to have algorithms which determine which curve in a family passes through an arbitrary point (T,P). Algorithms to accomplish this are given in Table 3.

---

[a] The X,Y coordinates have been scaled to USAF SKEW-T, log P diagram DOD-WPC-9-16-1. See [1].

[b] The bisection method is a numerical technique which decreases the difference between the upper and lower estimates by a factor of ½ per iteration.

Table 1: SKEW-T Algorithms

FAMILY	PARAMETER	ALGORITHM
Dry Adiabat	θ potential temperature	$T_{DA}(T,P) = \theta\left(\frac{P}{1000}\right)^{.288}$
Mixing Ratio	M Mixing ratio	T is in Kelvin, T = C + 273.16 $T_{MR}(W,P) = 10^{(a\log_{10}m+b)} + c + d(m^f + g)^2$ a = 0.0498646455 b = 2.4082965 c = 280.23475 d = 38.9114 f = 0.0915 g = -1.2035 M = W^*P / (622 + W)
Saturation Adiabat	θ_S the temperature at 1000 mb	$T_{SA}(\theta_S,P) = T_1 + \sum_{i=1}^{12} T_i^*$ T_1 = 253.16 K $T_i^* = \frac{120}{2^i} SIGN\left[a\exp\left\{\frac{bW(T_i,P)}{T_i}\right\} - T_i\left(\frac{1000}{P}\right)^{0.288}\right]$ $T_i = T_{i-1} + T_{i-1}^*$ $a = \theta_S$ b = -2.6518986 $W(T,P) = \frac{622ESAT(T)}{P - ESAT(T)}$ ESAT(T) = 10 ^ (23.832241 – 5.02808*$\log_{10}$(T) – 1.3816E-7 * 10 ^ (11.344 – 0.0303998*T) + 8.1328E-3 * 10 ^ (3.49149-1302.8844/T) – 2949.076/T) T is in Kelvin (K = C + 273.16) ESAT is from Nordquist[iii] The SIGN function is +1 or -1 corresponding to the algebraic sign of the argument.

Table 2: Temperature and error on Selected Saturation Adiabats at Selected Pressures

Pressure (mb)	Temperature (°C)	Error (°C)	Pressure (mb)	Temperature (°C)	Error (°C)
1000.0	40.0000				
701.5	29.9877	0.0122			
490.7	19.9536	0.0463			
344.7	9.9194	0.0805			
245.4	-0.1733	-0.1733			
179.6	-10.2221	-0.2221			
1000.0	30.0000		1000.0	-10.0000	
733.0	19.9829	0.0170	849.0	-20.0073	-0.0073
544.0	9.9633	0.3660	726.0	-29.9829	0.0170
412.4	-0.0561	-0.0561	621.0	-40.0756	-0.0756
321.4	-10.0756	-0.0756	531.2	-50.0512	-0.0512
257.7	-20.1538	-0.1538	452.2	-60.0415	-0.0415
212.0	-30.2612	-0.2612	382.4	-70.0463	-0.0463
177.6	-40.3247	-0.3427	266.9	-89.9975	-0.0024
1000.0	20.0000		1000.0	0.0000	
770.0	9.9780	0.0219	833.0	-9.9731	0.0268
606.0	-0.0561	-0.0561	703.0	-19.9926	0.0073
489.0	-10.0463	-0.0463	599.0	-29.9829	0.0170
403.0	-20.1245	-0.1245	511.0	-40.1196	-0.1196
338.0	-30.1879	-0.1879	436.4	-50.1391	-0.1391
286.4	-40.2368	-0.2368	371.3	-60.1293	-0.1293
243.5	-50.2709	-0.2709	314.0	-70.1196	-0.1196
206.8	-60.2612	-0.2612	263.5	-80.0952	-0.0952
174.7	-70.2661	-0.2661	219.1	-90.0854	-0.0854
1000.0			1000.0		
805.0	-0.0415	-0.0415	856.8	-30.0122	-0.0122
663.0	-9.9877	0.0122	734.8	-40.0170	-0.0170
554.0	-20.0952	-0.0952	628.6	-50.0366	-0.0366
470.0	-30.0268	-0.0268	535.3	-60.0268	-0.0268
400.0	-40.1196	-0.1196	452.6	-70.0170	-0.0170
341.0	-50.1538	-0.1538	380.0	-80.0073	-0.0073
289.9	-60.1586	-0.1586	316.0	-89.9829	0.0170
245.1	-70.1489	-0.1489			
205.7	-80.1098	-0.1098			
171.0	-90.1147	-0.1147			

Table 3: Determining a Curve through a Given Point

Family	Parameter for curve passing through (T,P)
Dry adiabat	$\theta = T\left(\frac{1000}{P}\right)^{0.288}$
Mixing ratio	$W = \frac{622ESAT(T)}{P - ESAT(T)}$
Saturation adiabat	$\theta_S = \frac{T\left(\frac{1000}{P}\right)^{0.288}}{\exp\left(\frac{bW(T,P)}{T}\right)}$b = -2.6518986

5. Algorithms for Selected Meteorological Quantities

Several meteorological quantities which are usually manually derived from an analysis of a SKEW-T were selected tor discussion. Algorithms are presented for computing these meteorological quantities. The selection of symbols is somewhat different than is customary because of current symbol limitations on computers. But by referring to the List of Symbols, the reader will have no difficulty. Units are the same as those used on the SKEW-T.

5.1. *Mixing Ratio W*

The *mixing ratio* W is computed from the pressure P and the dew point temperature TD by using the function ESAT which is defined in Table 1.

$$W = \frac{622ESAT(T)}{P - ESAT(T)} \tag{4}$$

TD is in degrees Kelvin, the pressure P in millibars and W in grams of water per kilogram dry air. The *saturation mixing ratio* is obtained by using the dry bulb temperature in place of the dew point temperature.

5.2. *Relative Humidity FR*

The relative humidity is computed from the Temperature T and the dew point temperature TD by using ESAT. Both T and TD are in degrees Kelvin.

$$FR = 100\frac{ESAT(TD)}{ESAT(T)} \tag{5}$$

5.3. *Saturation Vapor Pressure ES and Actual Vapor Pressure E*

ESAT gives the saturation vapor pressure in millibars from the dry bulb temperature T, which is in degrees Kelvin

$$\text{ES} = \text{ESAT(T)} \tag{6}$$

The actual vapor pressure E is found by using the dew point temperature TD instead of T in (6)

5.4. *Potential Temperature θ*

The potential temperature is computed from the dry bulb temperature T in Kelvin and the pressure P in millibars.

$$\theta = T\left(\frac{1000}{P}\right)^{0.288} \tag{7}$$

5.5. *Wet Bulb Temperature and Wet Bulb Potential Temperature: TW and θW*

The wet bulb temperature is approximated by calculating the pseudo wet bulb temperature. The arguments are surface dew point temperature, surface temperature and pressure which are symbolized by TDS, TS and PS respectively. TDS and TS are in Kelvin and P is in millibars. First, a mixing ratio curve W, which passes through (TDS,PS) is determined. Again by referring to Table 3 we have

$$W = \frac{622ESAT(TDS)}{P-ESAT(TDS)} \tag{8}$$

Next, a dry adiabat which passes through (TS,PS) is determined. Again by referring to Table 3 we have

$$\theta = TS\left(\frac{PS}{1000}\right)^{0.288} \tag{9}$$

Two curves have now been specified: $T_{MR}(W,P)$ and $T_{DA}(W,P)$. The next step is to locate the pressure at which the curves intersect. This is done by an iterative procedure. An initial guess that the intersection pressure PI is equal to the surface pressure is made. A correction is computed and a revised guess is made. Then $(T_{MR} - T_{DA})^2$ is less than 0.0001 degrees, the process is terminated.

$$PI_1 = PS \tag{10}$$

$$PI_i = PI_{i-1} = P^*_{i-1} \tag{11}$$

$$P^*_k = P_k 2^{0.02(T_{MR}(W,P_k)-T_{DA}(\theta,P_k))} \tag{12}$$

It is found that six iterations were sufficient to compute PI to within 1 mb. Once the pressure and hence temperature at the intersection are known, a saturation adiabat through the intersection point (TI,PI) is found. Referring to Table 3 we have

$$\theta_s = \frac{TI\frac{1000}{PI}^{0.288}}{\exp\frac{bW(TI,PI)}{TI}} \tag{13}$$

Finally, by following this saturation adiabat to the surface pressure PD and to 1000 mb, we get the wet bulb temperature TW and the wet bulb potential temperature θW respectively

$$TW = T_{SA}(\theta_s, PS) \tag{14}$$

$$\theta W = T_{SA}(\theta_s, 1000) \tag{15}$$

5.6. *The Pseudo Wet Bulb Temperature and Pseudo Wet Bulb Potential Temperature TPW and θPW*

Refer to the wet bulb temperature and wet bulb potential temperature above.

5.7. *The equivalent potential Temperature θ_e*

The equivalent potential temperature is computed from the same quantities used to compute the wet bulb temperature, i.e. the surface pressure, dew point temperature, and actual temperature. First compute the wet bulb temperature TW. The equivalent potential temperature can then be computed by the same process used to determine the parameter θ_S of a saturation adiabat through (TW,PS). Referring to Table 3, we have

$$\theta_e = \frac{TW\left(\frac{1000}{PS}\right)^{0.288}}{\exp\left(\frac{bW(TW,PS)}{TW}\right)} \tag{16}$$

5.8. The Pseudo Equivalent Temperature TE

First the equivalent potential temperature θ_e is computed. The pseudo equivalent temperature is then given by

$$TE = \theta_e \left(\frac{PS}{1000}\right)^{0.288} \tag{17}$$

5.9. Thickness of a Layer Z

It is assumed that the temperature and dew point temperature are known at N distinct, decreasing pressures. Thicknesses are computed in meters from the surface. The trapezoidal rule is used to integrate

$$Z = \frac{R}{0.98}\int_{\ln(PT)}^{\ln(PS)} \left[T + \frac{0.6078*W*T}{1000+W}\right] dlnP \tag{18}$$

See Table 1 for a definition of W(T,P). Rewriting Eq. (18) and noticing that W << 1000 gives Eq. (19), which is used to compute Z.

$$Z = 29.2857\left[\begin{array}{c} \frac{T_1+T_2+6.078\times10^{-6}(W_1T_1+W_2T_2)}{2}\ln\left(\frac{P_1}{P_2}\right) \\ +\frac{T_2+T_3+6.078\times10^{-6}(W_2T_2+W_3T_3)}{2}\ln\left(\frac{P_2}{P_3}\right) \\ +\frac{T_n+T_{n+1}+6.078\times10^{-6}(W_nT_n+W_{n+1}T_{n+1})}{2}\ln\left(\frac{P_n}{P_{n=1}}\right) \end{array}\right] \tag{19}$$

5.10. The Lifting Condensation Level LCL

The lifting condensation level is computed in the same manner that PI was computed for the wet bulb temperature, using Eqs. (8), (9), (10), (11), and (12). (TI,PI) locate the LCL.

5.11. The Convective Condensation Level CCL

It is assumed that the temperature and dew point temperature are known at N distinct, decreasing pressures. The pressure at the top of the of the mixing ratio PM must be greater than P_n, the last pressure level. Since PM is bounded by P_1 and P_n, there is a K such that

$$P_k > PM \geq P_{k+1} \tag{20}$$

First the mean mixing ratio $\bar{W}$ in the P_1-PM layer is computed:

$$\bar{W} = \frac{\sum_{i=1}^{k-1}[W(T_i,P_i)+W(T_{i+1},P_{i+1})]\ln(P_1)-\ln(P_{i+1})}{2(\ln(P_1)-\ln(P_{k+1}))} + \frac{[W(T_k,P_k)+W(T_m,PM)]\ln(P_k)-\ln(PM)}{2(\ln(P_1)-\ln(PM))} \tag{21}$$

The intersection of $T_{MR}(\bar{W},P)$ and the curve defined by

$$T_S(P) = T_K - \frac{(T_{k+1}-T_k)(\ln(P)-\ln(P_k))}{\ln(P_k)-\ln(P_{k+1})} \tag{22}$$

(k is chosen such that $P_k \geq P \geq P_{k+1}$) defines the convective condensation level. This intersection can be found by first systematically comparing the difference between $T_{MR}(\bar{W}, P_i)$ and $T_S(P_i)$ until the smallest is found such that

$$T_{MR}(\bar{W}, P_i) - T_S(P_i) < 0 \tag{23}$$

and

$$T_{MR}(\bar{W}, P_{i+1}) - T_S(P_{i+1}) < 0 \tag{24}$$

A bisection method is used to determine PC, the pressure at the CCL. An initial guess PC_1 is made, tested to see if $T_{MR}(\bar{W}, PC_1)$ equals $T_S(PC_1)$, and if not, corrected.

$$PC_1 = 0.5(Pi + P_{i-1}) \tag{25}$$

$$PC_j = PC_{j-1} + P^*_{j-1} \quad \text{(corrector)} \tag{26}$$

$$P^*_k = \frac{P_i+P_{i+1}}{2^k} SIGN(T_{MR}(\bar{W}, PC_k) - T_S(P_k)) \tag{27}$$

Ten corrections are made.

5.12. The Convective Temperature CT

First, the pressure PC at the convective condensation level is computed. The temperature at the CCL, TC, is computed from PC and $\overline{W}$:

$$TC = T_{MR}(\overline{W}, PC) \tag{28}$$

A dry adiabat is determined:

$$\theta = TC\left(\frac{1000}{PC}\right)^{.288} \tag{29}$$

Finally, the convective temperature CT is computed from θ and the surface pressure PS:

$$CT = \theta\left(\frac{PS}{1000}\right)^{.288} \tag{30}$$

6. Applications

The algorithms are useful for data reduction purposes. The memory and speed requirements are not excessive and most computations can be carried out successfully on a programmable calculator. In addition to data analysis, the algorithms are useful for generating backgrounds for the presentation of data. An example of a computer generated background and plotted sounding is given in Fig. 1. Computation of selected meteorological quantities from the plotted sounding in Fig. 1 is presented in Table 4. A table of CCL temperatures, pressures and heights was computed using an arbitrary decrement of -25 mb for the pressure at the top of the mixing layer.

Table 4: An Example of a Vertical Sounding

Pressure	1013	953	950	942	920	843	777	745	691	620	333	210
Temperature	20.4	18.2	19	18.2	14.4	5.8	-0.7	-0.1	-5.5	-12.3	-20.1	-25.5
Dew Point	18.2	14.4	6	-0.8	-0.6	-5.2	-12.7	-20.1	-25.5	-30.3	-28.1	-32.5
Height	0	527	554	626	827	1554	2213	2549	3145	3985	8664	12047
Potential Temp	19.3	22.3	23.3	23.2	21.4	19.9	19.8	24.0	24.6	26.2	74.2	115.0
Relative Humidity	87	78	43	28	36	45	40	21	19	21	49	52
Mixing Ratio	13.09	10.88	6.18	3.82	3.98	3.08	1.85	1.04	0.69	0.50	1.14	1.19
Saturation VP	23.9	20.9	21.9	20.9	16.4	9.2	5.8	6.1	4.1	2.4	1.2	0.8
Vapour Pressure	20.9	16.4	9.3	5.8	5.8	4.1	2.3	1.2	0.8	0.5	0.6	0.4
Wet Bulb Temp	19.0	15.9	11.7	8.7	7.1	1.1	-4.7	-5.9	-10.2	-15.7	-22.1	-27.4
Potential W Bulb T	18.5	17.8	13.8	11.3	10.8	9.1	7.6	8.4	8.2	8.6	23.9	31.1
Equiv Pot Temp	56.3	53.5	41.4	34.7	33.1	29.0	25.4	27.3	26.8	27.8	78.5	120.2

Lifting Condensation Level (LCL): Temperature 17.9, Pressure 983, Height 260

Mixing Layer		Convective Condensation Level			
Pressure	Height	Pressure	Height	Convective Temp	Mean Mixing Ratio
988	215	931	726	23.4	12.63
963	437	928	758	23.1	12.17
938	662	910	915	22.3	10.63
913	891	881	1183	21.8	8.89
888	1121	862	1372	21.4	7.8
863	1359	846	1522	21.0	7.03
838	1602	831	1670	21.0	6.43
813	1847	816	1814	21.0	5.92
788	2099	803	1951	21.0	5.47
763	2358	789	2084	21.0	5.06

Units: temperature °C, pressure millibar, mixing ratio g/kg, height metres

Note: values have been corrected from the original document.

7. Acknowledgements

Many individuals and groups contributed stimulating discussion and valuable time in assisting the author on this study and it is impossible to name them all. Deep appreciation is extended to the National Center for Atmospheric Research, which is sponsored by the National Science Foundation, for computer time used in this research. Mr. Walter S. Nordquist, who provided the impetus for this study, as well as many suggestions along the way and a critical reading of the manuscript, deserves much of the credit for this work. Finally, Mr. Alex Blomerth provided much administrative assistance, without which this study would not have been possible.

8. Addenda by Harold Reynolds

8.1. Corrections to Original Manuscript

The original definition of θ_e, Equation (16), does not agree with the standard definition,

$$\theta_e = \theta \; exp\left(\frac{LW(TD,P)}{c_P \; T_{SAT}}\right) \tag{34}$$

where L is the latent head of condensation of water, c_P is the specific heat of air at constant pressure, and T_{SAT} is the temperature at which the air parcel becomes saturated when lifted adiabatically. This temperature can be readily found using the Lifting Condensation Level subroutine to find the pressure of the LCL, followed by finding the temperature on the appropriate dry adiabat at that pressure.

Table 4 of the original manuscript has been revised to reflect the values I found when running the thermodynamic subroutines on the sample sounding.

8.2. Thermodynamic Functions Visual Basic Code

Listed below are the various thermodynamic functions discussed in the paper, translated into Visual Basic from the original Fortran. Like with any code, *you must check and verify the correctness of the code yourself before using it for any project*. I was able to reproduce the results from the first table in Table 4, which is a hopeful sign.

```
Option Explicit

'Thermodynamic chart software package.
'Developed by G. S. Stipanuk, Atmospheric Sciences
Laboratory, White Sands
'Missile Range, New Mexico, 88002. Presented in
"Algorithms for Generating
'a SKEW-T, log P diagram and computing selected
meteorological quantities",
'U.S. Government publication ECOM-5515, published
October, 1973, and available
'on microfiche from NTIS.
'Typed in (in FORTRAN) by Harold Reynolds, March
15, 1991.
'Translated into Visual Basic by Harold Reynolds,
June 28 - July 2, 2009.

'The following subroutines approximate a
thermodynamic chart.
'T is the temperature in Kelvin. Scalar in all
functions except Z and CCL.
'TD is the dew point temperature.     Ditto.
'P is the pressure in millibars.      Ditto.
'TDS, TS, and PS are TD, T, and P at the surface.
'WBAR is the mean mixing ratio.
'O is really a theta.
'Soundings must be ordered by decreasing pressure.

'Harold's note: function names are capitalized,
variables are not.
'LEGAL STUFF: To the best of my knowledge, these
routines produce the correct
'results. However, YOU MUST VERIFY THEIR
CORRECTNESS BEFORE USING THESE
'FUNCTIONS FOR ANY PROJECT! IF SOMETHING IS WRONG,
I WOULD VERY MUCH LIKE
'TO CORRECT IT! YOU HAVE BEEN WARNED!

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function ALCL(tds As Double, ts As Double, ps As
Double) As Double
'Computes the pressure at the lifting condensation
level.
'tds, ts in K, ps, alcl in mb. ABS = absolute
value.
  Dim aw As Double, ao As Double, pi As Double, x
As Double
  Dim i As Integer

  aw = W(tds, ps)
  ao = O(ts, ps)
  pi = ps
  For i = 1 To 10
    x = 0.02 * (TMR(aw, pi) - TDA(ao, pi))
    If Abs(x) < 0.01 Then Exit For
    pi = pi * 2 ^ x
  Next i
  ALCL = pi
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function CCL(pm As Double, p() As Double, t() As
Double, td() As Double, _
  wbar As Double, N As Integer) As Double
'Computes pressure at convective condensation
level.
'N is the number of levels in the sounding. K is
the last level below pm.
'PM is pressure at top of mixing layer.
'CCL and p in mb, t in Kelvin, war in g vapour/kg
dry air.
  Dim t() As Double, td() As Double, p() As Double,
tq As Double, x As Double
  Dim del As Double, pc As Double, a As Double
  Dim k As Integer, j As Integer, i As Integer, L
As Integer
  Dim FoundIt As Boolean

  wbar = 0
  k = 1
  Do While p(k) < pm
    k = k + 1
  Loop

  k = k - 1
  j = k - 1

  If j >= 1 Then
'Compute the average mixing ratio. Log is natural
logarithm
    For i = 1 To j
      L = i + 1
      wbar = (W(td(i), p(i)) + W(td(L), p(L))) *
Log(p(i) / p(L)) + wbar
    Next i
  End If

  L = k + 1
  tq = td(k) + (td(L) - td(k)) * Log(pm / p(k)) /
Log(p(L) / p(k))
  wbar = wbar + (W(td(k), p(k)) + W(tq, pm)) *
Log(p(k) / pm)
  wbar = wbar / (2 * Log(p(1) / pm))

'Find the level at which tmr - ts changes sign. TS
is sounding temp.
```

```
  FoundIt = False
  For i = 1 To N
    If TMR(wbar, p(i)) + 273.16 >= 0 Then
      FoundIt = True
      Exit For
    End If
  Next i

'Not found, exit with CCL = 0
  CCL = 0
  Exit Function

'Set up bisection routine
  L = i - 1
  del = p(L) - p(i)
  pc = p(i) + 0.5 * del
  a = (t(i) - t(L)) / Log(p(L) / pc) + 273.16

  For i = 1 To 10
    del = del / 2
    x = TMR(wbar, pc) - t(L) - a * Log(p(L) / pc) +
273.16
'The SIGN(x,y) function is a FORTRAN intrinsic that
replaces the sign of x
'with that of y. I
'had to make a separate function for it here.
    pc = pc + SIGN(del, x)
  Next i

  CCL = pc
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function CT(wbar As Double, pc As Double, ps As
Double) As Double
'Computes the convective temperature.
'Wbar in g/kg, pc, ps in mb.
  Dim tc As Double, ao As Double

  tc = TMR(wbar, pc) + 273.16
  ao = O(tc, pc)
  CT = TDA(ao, ps)
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function ESAT(t As Double) As Double
'Computes the saturation vapour pressure over water
at temperature t.
'ESAT in mb, t in K. Log to base 10 is needed for
this function.
  Dim a0 As Double, a1 As Double, a2 As Double

  a0 = 23.832241 - 5.02808 * Log10(t)
  a1 = 0.00000013816 * 10 ^ (11.344 - 0.0303998 *
t)
  a2 = 0.0081328 * 10 ^ (3.49149 - 1302.8844 / t)

  ESAT = 10 ^ (a0 - a1 + a2 - 2949.076 / t)
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function FR(t As Double, td As Double) As Double
'Computes relative humidity. FR in percent, t, td
in Kelvin.

  FR = ESAT(td) / ESAT(t) * 100
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function O(t As Double, p As Double) As Double
'Computes the dry adiabat through (t,p)
'O and T in K, p in mb.
  O = t * (1000 / p) ^ 0.288
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function OE(tds As Double, ts As Double, ps As
Double) As Double
'Computes the potential equivalent / pseudo-
equivalent temperature.
'tds, ts, OE in K, ps in mb.
'NOTE: The commented formula is the one from the
paper and gives wrong results!
'I have used instead the formula from Holton, p.
331, with q as the mixing
'ratio of the parcel and T as the saturation
temperature (temperature at the LCL).
  Dim alift As Double, olift As Double, tlift As
Double

  alift = ALCL(tds, ts, ps)
  olift = O(ts, ps)
  tlift = TDA(olift, alift) + 273.16
  OE = O(ts, ps) * Exp(2.6518986 * W(tds, ps) /
tlift) - 273.16

'This is the code which gives the SATURATED OE
'  atw = TW(tds,ts,ps) + 273.16
'  OE = OS(atw,1000) -273.16
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function OS(t As Double, p As Double) As Double
'Computes saturation adiabat through (t,p)
'OS, t in K, p in millibars (mb)

  OS = t * (1000 / p) ^ 0.288 / Exp(-2.6518986 *
W(t, p) / t)
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function OW(tds As Double, ts As Double, ps As
Double) As Double
'Computes potential wet bulb temperature.
'tds, ts and OW in K, ps in mb.
  Dim atw As Double, aos As Double

  atw = TW(tds, ts, ps) + 273.16
  aos = OS(atw, ps)
  OW = TSA(aos, 1000)
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function TDA(O As Double, p As Double) As Double
'Computes temperature on a dry adiabat o (theta) at
pressure p
'o, TDA in K, p in mb.

  TDA = O * (p / 1000) ^ 0.288 - 273.16
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function TE(tds As Double, ts As Double, ps As
Double) As Double
'Computes equivalent temperature
'tds, td, TE in K, ps in mb.
  Dim aoe As Double

  aoe = OE(tds, ts, ps) + 273.16
  TE = TDA(aoe, ps)
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function TMR(W As Double, p As Double) As Double
```

```
'Computes temperature on mixing ratio w at pressure
p.
'TMR in C, w in g/kg dry air, p in millibars.
  Dim x As Double

  x = Log10(W * p / (622 + W))
  TMR = 10 ^ (0.0498646455 * x + 2.4082965) -
280.23475 + 38.9114 * _
    ((10 ^ (0.0915 * x) - 1.2035) ^ 2)
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function TSA(OS As Double, p As Double) As Double
'Computes temperature on saturated adiabat os at
pressure p.
'SIGN(a,b) replaces the algebraic sign of a with
that of b.
  Dim a As Double, tq As Double, d As Double, x As
Double
  Dim i As Integer

  a = OS
  tq = 253.16
  d = 120
  For i = 1 To 12
    d = d / 2
'If the temperature difference x is small, exit the
loop
    x = a * Exp(-2.6518986 * W(tq, p) / tq) - tq *
(1000 / p) ^ 0.288
    If Abs(x) <= 0.0000001 Then Exit For
    tq = tq + SIGN(d, x)
  Next i
  TSA = tq - 273.16
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function TW(tds As Double, ts As Double, ps As
Double) As Double
'Computes wet bulb temperature.
'tds, ts and TW in K, ps in mb.
  Dim i As Integer
  Dim aw As Double, ao As Double, pi As Double, x
As Double
  Dim ti As Double, aos As Double

  aw = W(tds, ps)
  ao = O(ts, ps)
  pi = ps
  For i = 1 To 10
    x = 0.02 * (TMR(aw, pi) - TDA(ao, pi))
    If Abs(x) <= 0.01 Then Exit For
    pi = pi * 2 ^ x
  Next i
  ti = TDA(ao, pi) + 273.16
'The intersection has been found, now find a
saturated adiabat through it.
  aos = OS(ti, pi)
  TW = TSA(aos, ps)
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function W(t As Double, p As Double) As Double
'Computes the mixing ratio line through (t,p).
'T is in K, p in mb, W in g water /kg dry air.
  Dim x As Double

  x = ESAT(t)
  W = 622 * x / (p - x)
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function Z(pt As Double, p() As Double, t() As
Double, td() As Double, _
  N As Integer) As Double
'Computes thickness in metres from p(1) to pt.
  Dim i As Integer, j As Integer
  Dim a1 As Double, a2 As Double, Z1 As Double

  Z1 = 0
  For i = 1 To N
    j = i + 1
    If pt >= p(j) Then Exit For
    a1 = t(j) * (1 + 0.0006078 * W(td(j), p(j)))
    a2 = t(i) * (1 + 0.0006078 * W(td(i), p(i)))
    Z1 = Z1 + 14.64285 * (a1 + a2) * Log(p(i) /
p(j))
  Next i
    a1 = t(j) * (1 + 0.0006078 * W(td(j), p(j)))
    a2 = t(i) * (1 + 0.0006078 * W(td(i), p(i)))
    Z = Z1 + 14.64285 * (a1 + a2) * Log(p(i) / pt)
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function Log10(ByVal x As Double) As Double
'Computes logarithm to base 10

  Log10 = Log(x) / Log(10)
End Function

'-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=
Function SIGN(x As Double, y As Double) As Double
'Replaces the sign of x with that of y. This is
used to mimic the intrinsic
'function SIGN in Fortran.

  If y < 0 Then
    SIGN = -Abs(x)
  Else
    SIGN = Abs(x)
  End If
End Function
```

9. Bibliography

1. "Radiosonde Code (Standards and Procedures for the Coding of Radiosonde Reports)," Federal Meteorological Handbook No.4, Superintendent of Documents, US Government Printing Office, Washington, D.C. (A thorough explanation of the radiosonde code is presented.)
2. Air Weather Service (MATS), 1961, "Use of the SKEW-T, log p, DIAGRAM in Analysis and Forecasting, Volume 1 Radiosonde Analysis," AWS Manual No. 105-124, Scott Air Force Base, Illinois 62225. (Although this manual has been updated, it is still useful.)
3. Air Weather Service (MAC), 1969, "Use of the SKEW-T, log p, DIAGRAM in Analysis and Forecasting, Volume 1 Radiosonde Analysis," AWS Manual No. 105-124, Scott Air Force Base, Illinois 62225. (AD 695 603). (This version contains a great deal more material than the 1961 manual.)
4. Nordquist, W.S., 1969, "Determination of the Lifting Condensation Level," ECOM-5282, Atmospheric Sciences Laboratory, US Army Electronics Command, White Sands Missile Range, New Mexico 88002 (AD 700 953). (Another method for computing the LCL is presented.)
5. Johnson, N.L., 1969, "Program Description for the Automatic Temperature-Pressure Data on a Skew-T, log p DIAGRAM," ECOM-5266, Atmospheric Sciences Laboratory, US Army Electronics Command, White Sands Missile Range, New Mexico 88002 (AD 697 789) (A FORTRAN program is presented for plotting profiles in aSKEW - T format. The method applies to a particular hardware configuration and would not be useful for other systems.)
6. Mahlmar, J.D., and W. Kamm, 1965, "Development of Computer Programs for Computation of Montgomery Stream Functions and Plotting of Thermodynamic Diagrams," Colorado State University Technical Notes, Department of Atmospheric Sciences, Fort Collins, Coloradci,80521. (Machine procedures are developed for computation of isentropic analysis parameters and for plotting of tephigrams, both in terms of derived basic equations. This paper gives a FORTRAN program listing. Input and output is limited to punched cards.)
7. Prosser, Norman E.; and Donald S. Foster, 1966, "Upper Air Sounding Analysis by Use of an Electronic Computer," Journal of Applied Meteorology,**5,** pp. 296-300. (This is an interesting article on upper air sounding analysis used by the Severe Local Storm Forecasting Unit.)
8. Herlofson, N., 1947, "The T, log p - DIAGRAM with SKEW Coordinate Axis," Meteorolgiske Annaler BD. 2, NR. 10, PP. 310-342. (The original idea of a SKEW-T, log p diagram is presented in this paper.)

End Notes

---

[i] Department of Defence, 1960, “USAF SKEW-T, log p DIAGRAM”, DOD-WPC-9-16-1, Aeronautical Chart and Information Center, United States Air Force, St. Louis, Missouri, 63118.

[ii] List, R.J. (Editor), 1958, Smithsonian Meteorological Tables, Smithsonian Institute, Washington, DC.

[iii] Nordquist, W.S., 1973, “Numerical Approximation of Selected Meteorological Parameters for Cloud Physics Problems”, ECOM-3475, Atmospheric Sciences Laboratory, US Army Electronics Command, White Sands Missile Range, New Mexico, 88002.