Algorithms for Generating a Skew-T, Log P Diagram
and Computing Selected Meteorological Quantities

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October 1973
Report ECOM-5515

With Updates by Harold Reynolds
April 7, 1991
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1. Abstract

This paper discusses numerical methods of computing meteorological quantities which are usually manually derived from analysis on a SKEW-T, log p diagram. The numerical methods used were selected for their simplicity and accuracy. A mathematical characterization of the SKEW-T and algorithm for computing several meteorological quantities are presented.

2. Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCL</td>
<td>Convective condensation level</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Heat capacity of air at constant pressure</td>
</tr>
<tr>
<td>CT</td>
<td>Convective temperature</td>
</tr>
<tr>
<td>E</td>
<td>Actual vapor pressure</td>
</tr>
<tr>
<td>ES</td>
<td>Saturation vapor pressure</td>
</tr>
<tr>
<td>FR</td>
<td>Relative humidity</td>
</tr>
<tr>
<td>i, j, k</td>
<td>Indexes</td>
</tr>
<tr>
<td>L</td>
<td>Latent heat of vaporization of water</td>
</tr>
<tr>
<td>LCL</td>
<td>Lifting condensation level</td>
</tr>
<tr>
<td>M</td>
<td>Saturation vapor pressure over water</td>
</tr>
<tr>
<td>P</td>
<td>Pressure</td>
</tr>
<tr>
<td>$P'$</td>
<td>Pressure correction</td>
</tr>
<tr>
<td>PB</td>
<td>Pressure at the bottom of a layer</td>
</tr>
<tr>
<td>PC</td>
<td>Pressure at the convective condensation level</td>
</tr>
<tr>
<td>PI</td>
<td>Pressure at the intersection</td>
</tr>
<tr>
<td>PM</td>
<td>Pressure at the top of the mixing layer</td>
</tr>
<tr>
<td>PS</td>
<td>Surface pressure</td>
</tr>
<tr>
<td>R</td>
<td>Gas Constant</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>$T^*$</td>
<td>Temperature correction</td>
</tr>
<tr>
<td>TD</td>
<td>Dew point temperature</td>
</tr>
<tr>
<td>TDS</td>
<td>Dew point temperature at the surface</td>
</tr>
<tr>
<td>$T_{DA}$</td>
<td>Temperature on a dry adiabatic curve</td>
</tr>
<tr>
<td>$T_{M}$</td>
<td>Temperature at the top of the mixing layer</td>
</tr>
<tr>
<td>$T_{MR}$</td>
<td>Temperature on a mixing ratio curve</td>
</tr>
<tr>
<td>$T_{SA}$</td>
<td>Temperature on a saturation adiabat curve</td>
</tr>
<tr>
<td>TW</td>
<td>Wet bulb temperature</td>
</tr>
<tr>
<td>$\bar{W}$</td>
<td>Mean mixing ratio</td>
</tr>
<tr>
<td>X</td>
<td>Coordinate</td>
</tr>
<tr>
<td>Y</td>
<td>Coordinate</td>
</tr>
<tr>
<td>Z</td>
<td>Thickness of a layer</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Potential temperature</td>
</tr>
<tr>
<td>$\theta_e$</td>
<td>Equivalent potential temperature</td>
</tr>
<tr>
<td>$\theta_S$</td>
<td>Parameter for saturation adiabat</td>
</tr>
</tbody>
</table>

3. Introduction

The increasing availability of computing facilities, programmable calculators, and mini-computers allows many of the computations currently performed by manual graphics to be done by computer. This paper discusses numerical methods of computing meteorological quantities which are usually manually derived from analysis on a SKEW-T, log p diagram (or SKEW-T). The numerical methods used were selected for their simplicity and accuracy. A mathematical characterization of the SKEW-T and algorithms for computing several meteorological variables are presented. Finally, a discussion of the application of these methods and a FORTRAN program listing to accomplish the computations are included.
4. The SKEW-T, log p Diagram

The SKEW-T, log p diagram is a thermodynamic chart with five families of curves, five types of scales and three data blocks. Various numerical information is also tabulated on the SKEW-T. This paper is concerned chiefly with the five families of curves which are associated with pressure, temperature, dry adiabat, saturation adiabat, and mixing ratio.

The first two families of curves, temperature and pressure, are used to locate points on the chart. An arbitrary coordinate system has been selected to measure distances. Let the origin correspond to the point at a temperature of 0°C (centigrade) and a pressure of 1000 mb (millibars). Take the X direction to be parallel to the pressure lines (horizontal), with positive X to the right. The point at a temperature of 1°C and a pressure of 1000 mb is on the positive side of the origin. The Y direction is perpendicular to the X direction. Positive Y is towards lower pressures (up). A point on the chart which is specified by its temperature and pressure may be transformed to X, Y coordinates by Equations (1) and (2). The components of the X, Y system are given in inches:\(^a\)

\[
X = 0.1408T - 10.53975 \log_{10} P + 31.61923 \\
Y = -11.5 \log_{10} P + 34.5
\]  

The remaining three families of curves, TWR, TSA, and TDA, are given in Table 1. The temperatures are specified as a function of pressure and a parameter, the parameter serving as a means of specifying a particular curve of the family.

The temperature \( T \) at an arbitrary pressure on a saturation adiabat is determined by the bisection method.\(^b\) The temperature is assumed to lie in the range -80°C to 40°C. An initial guess of -20°C is made and the correction \( T^* \) is computed. The correction term decreases by a factor of \( \frac{1}{2} \) after each correction. Terminating after 13 corrections gave satisfactory results. The algorithm for computing the temperature on a saturation adiabat is based on Equation 3:

\[
\theta = \theta_e \exp \left( \frac{-LW}{C_p T} \right)
\]

The latent heat of vaporization \( L \) and the heat capacity of air at constant pressure \( C_p \) are considered constant. Since it is not known how accurately the saturation adiabat could be determined from Eq. (3), Table 2 was constructed using List\(^ii\) as a standard. The temperature on an arbitrary mixing ratio curve \( W \) is computed by first computing the saturation vapor pressure \( M \). An approximation to the inverse saturation vapor pressure function is then used to compute the temperature.

In addition to the algorithms which generate the curves for each family, it is necessary to have algorithms which determine which curve in a family passes through an arbitrary point \((T,P)\). Algorithms to accomplish this are given in Table 3.

\(^a\)The X, Y coordinates have been scaled to USAF SKEW-T, log P diagram DOD-WPC-9-16-I. See [1].

\(^b\)The bisection method is a numerical technique which decreases the difference between the upper and lower estimates by a factor of \( \frac{1}{2} \) per iteration.
<table>
<thead>
<tr>
<th>FAMILY</th>
<th>PARAMETER</th>
<th>ALGORITHM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry Adiabat</td>
<td>θ potential</td>
<td>( T_{DA}(T, P) = \theta \left( \frac{P}{1000} \right)^{288} )</td>
</tr>
<tr>
<td></td>
<td>temperature</td>
<td></td>
</tr>
<tr>
<td>Mixing Ratio</td>
<td>M Mixing ratio</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T is in Kelvin,</td>
<td>T = C + 273.16</td>
</tr>
<tr>
<td></td>
<td>( T_{MR}(W, P) = 10^{(a\log_{10}m+b)} + c + d(m^f + g)^2 )</td>
<td>a = 0.0498646455</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b = 2.4082965</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c = 280.23475</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d = 38.9114</td>
</tr>
<tr>
<td></td>
<td></td>
<td>f = 0.0915</td>
</tr>
<tr>
<td></td>
<td></td>
<td>g = -1.2035</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M = WP / (622 + W)</td>
</tr>
<tr>
<td>Saturation Adiabat</td>
<td>( \theta_S )</td>
<td>T_1 = 253.16 K</td>
</tr>
<tr>
<td></td>
<td>the temperature</td>
<td>T_i = \frac{120}{2^i} \cdot \text{SIGN} \left[ \exp \left( \frac{bW(T_i, P)}{T_i} \right) - T_i \left( \frac{1000}{P} \right)^{0.288} \right]</td>
</tr>
<tr>
<td></td>
<td>at 1000 mb</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T_i = T_i-1 +</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T_i__1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a = \theta_S</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b = -2.6518986</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W(T, P) = \frac{622ESAT(T)}{P - ESAT(T)}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ESAT(T) = 10^{(23.832241 - 5.02808 \times \log_{10}(T) - 1.3816E-7 * 10^\backslash 1302.8844/T)} - 2949.076/T</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T is in Kelvin (K = C + 273.16)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ESAT is from Nordquist \text{iii}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The \text{SIGN} function is +1 or -1 corresponding to the algebraic sign of the argument.</td>
<td></td>
</tr>
</tbody>
</table>

\text{iii} Nordquist, R. (1970).
Table 2: Temperature and error on Selected Saturation Adiabats at Selected Pressures

<table>
<thead>
<tr>
<th>Pressure (mb)</th>
<th>Temperature (°C)</th>
<th>Error (°C)</th>
<th>Pressure (mb)</th>
<th>Temperature (°C)</th>
<th>Error (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000.0</td>
<td>40.0000</td>
<td>0.0122</td>
<td>1000.0</td>
<td>-10.0000</td>
<td>-0.0073</td>
</tr>
<tr>
<td>701.5</td>
<td>29.9877</td>
<td>0.0463</td>
<td>849.0</td>
<td>-20.0073</td>
<td>0.0170</td>
</tr>
<tr>
<td>490.7</td>
<td>19.9536</td>
<td>0.0805</td>
<td>726.0</td>
<td>-29.9829</td>
<td>-0.0756</td>
</tr>
<tr>
<td>344.7</td>
<td>9.9194</td>
<td></td>
<td>621.0</td>
<td>-40.0756</td>
<td>-0.0512</td>
</tr>
<tr>
<td>245.4</td>
<td>-0.1733</td>
<td></td>
<td>531.2</td>
<td>-50.0512</td>
<td>-0.0415</td>
</tr>
<tr>
<td>179.6</td>
<td>-10.2221</td>
<td></td>
<td>452.2</td>
<td>-60.0415</td>
<td>-0.0463</td>
</tr>
<tr>
<td>1000.0</td>
<td>30.0000</td>
<td>0.0170</td>
<td>382.4</td>
<td>-70.0463</td>
<td>-0.0024</td>
</tr>
<tr>
<td>733.0</td>
<td>19.9829</td>
<td>0.0561</td>
<td>266.9</td>
<td>-89.9975</td>
<td>-0.0073</td>
</tr>
<tr>
<td>544.0</td>
<td>9.9633</td>
<td>0.0561</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>412.4</td>
<td>-0.0561</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>321.4</td>
<td>-10.0756</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>257.7</td>
<td>-20.1538</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>212.0</td>
<td>-30.2612</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>177.6</td>
<td>-40.3247</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>1000.0</td>
<td>20.0000</td>
<td>0.0219</td>
<td>1000.0</td>
<td>0.0000</td>
<td>-0.0060</td>
</tr>
<tr>
<td>770.0</td>
<td>9.9780</td>
<td>0.0561</td>
<td>833.0</td>
<td>-9.9731</td>
<td>-0.1196</td>
</tr>
<tr>
<td>606.0</td>
<td>-0.0561</td>
<td></td>
<td>703.0</td>
<td>-19.9926</td>
<td>0.0073</td>
</tr>
<tr>
<td>489.0</td>
<td>-10.0463</td>
<td></td>
<td>599.0</td>
<td>-29.9829</td>
<td>0.0170</td>
</tr>
<tr>
<td>403.0</td>
<td>-20.1245</td>
<td></td>
<td>511.0</td>
<td>-40.1196</td>
<td>-0.1391</td>
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<tr>
<td>338.0</td>
<td>-30.1879</td>
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<td>436.4</td>
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<td>286.4</td>
<td>-40.2368</td>
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<td>371.3</td>
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<tr>
<td>243.5</td>
<td>-50.2709</td>
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<td>314.0</td>
<td>-70.1196</td>
<td>-0.1196</td>
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<tr>
<td>206.8</td>
<td>-60.2612</td>
<td></td>
<td>263.5</td>
<td>-80.0952</td>
<td>-0.0952</td>
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<tr>
<td>174.7</td>
<td>-70.2661</td>
<td></td>
<td>219.1</td>
<td>-90.0854</td>
<td>-0.0854</td>
</tr>
<tr>
<td>1000.0</td>
<td>-0.0415</td>
<td>-0.0415</td>
<td>1000.0</td>
<td>-30.0122</td>
<td>-0.0122</td>
</tr>
<tr>
<td>805.0</td>
<td>-9.9877</td>
<td>0.0122</td>
<td>856.8</td>
<td>-40.0170</td>
<td>-0.0170</td>
</tr>
<tr>
<td>663.0</td>
<td>-20.0952</td>
<td></td>
<td>734.8</td>
<td>-50.0366</td>
<td>-0.0366</td>
</tr>
<tr>
<td>554.0</td>
<td>-30.0268</td>
<td></td>
<td>628.6</td>
<td>-60.0268</td>
<td>-0.0268</td>
</tr>
<tr>
<td>470.0</td>
<td>-40.1196</td>
<td></td>
<td>535.3</td>
<td>-70.0170</td>
<td>-0.0170</td>
</tr>
<tr>
<td>400.0</td>
<td>-50.1538</td>
<td></td>
<td>452.6</td>
<td>-80.0073</td>
<td>-0.0073</td>
</tr>
<tr>
<td>341.0</td>
<td>-60.1586</td>
<td></td>
<td>380.0</td>
<td>-89.9829</td>
<td>-0.0170</td>
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<td>289.9</td>
<td>-70.1489</td>
<td></td>
<td>316.0</td>
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</tr>
<tr>
<td>245.1</td>
<td>-80.1098</td>
<td></td>
<td></td>
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</tr>
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<td>205.7</td>
<td>-90.1147</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Algorithms for Selected Meteorological Quantities

Several meteorological quantities which are usually manually derived from an analysis of a SKEW-T were selected for discussion. Algorithms are presented for computing these meteorological quantities. The selection of symbols is somewhat different than is customary because of current symbol limitations on computers. But by referring to the List of Symbols, the reader will have no difficulty. Units are the same as those used on the SKEW-T.

5.1. Mixing Ratio W

The mixing ratio $W$ is computed from the pressure $P$ and the dew point temperature $T_D$ by using the function $ESAT$ which is defined in Table 1.

$$ W = \frac{622ESAT(T)}{P - ESAT(T)} \quad (4) $$

$T_D$ is in degrees Kelvin, the pressure $P$ in millibars and $W$ in grams of water per kilogram dry air. The saturation mixing ratio is obtained by using the dry bulb temperature in place of the dew point temperature.

5.2. Relative Humidity $FR$

The relative humidity is computed from the Temperature $T$ and the dew point temperature $T_D$ by using $ESAT$. Both $T$ and $T_D$ are in degrees Kelvin.

$$ FR = 100 \frac{ESAT(T_D)}{ESAT(T)} \quad (5) $$

5.3. Saturation Vapor Pressure $ES$ and Actual Vapor Pressure $E$

$ESAT$ gives the saturation vapor pressure in millibars from the dry bulb temperature $T$, which is in degrees Kelvin

$$ ES = ESAT(T) \quad (6) $$

The actual vapor pressure $E$ is found by using the dew point temperature $T_D$ instead of $T$ in (6)

5.4. Potential Temperature $\theta$

The potential temperature is computed from the dry bulb temperature $T$ in Kelvin and the pressure $P$ in millibars.
\[ \theta = T \left( \frac{1000}{p} \right)^{0.288} \] (7)

5.5. Wet Bulb Temperature and Wet Bulb Potential Temperature: \( TW \) and \( \theta W \)

The wet bulb temperature is approximated by calculating the pseudo wet bulb temperature. The arguments are surface dew point temperature, surface temperature and pressure which are symbolized by TDS, TS and P respectively. TDS and TS are in Kelvin and P is in millibars. First, a mixing ratio curve \( W \), which passes through \( (TDS,PS) \) is determined. Again by referring to Table 3 we have

\[ W = \frac{622ESAT(TDS)}{P-ESAT(TDS)} \] (8)

Next, a dry adiabat which passes through \( (TS,PS) \) is determined. Again by referring to Table 3 we have

\[ \theta = TS \left( \frac{PS}{1000} \right)^{0.288} \] (9)

Two curves have now been specified: \( T_{MR}(W,P) \) and \( T_{DA}(W,P) \). The next step is to locate the pressure at which the curves intersect. This is done by an iterative procedure. An initial guess that the intersection pressure \( PI \) is equal to the surface pressure is made. A correction is computed and a revised guess is made. Then \( (T_{MR} - T_{DA})^2 \) is less than 0.0001 degrees, the process is terminated.

\[ PI_1 = PS \] (10)

\[ PI_l = PI_{l-1} = P_{l-1}^* \] (11)

\[ P_k^* = P_k 2^{0.02(T_{MR}(W,P_k) - T_{DA}(\theta, P_k))} \] (12)

It is found that six iterations were sufficient to compute \( PI \) to within 1 mb. Once the pressure and hence temperature at the intersection are known, a saturation adiabat through the intersection point \( (TI, PI) \) is found. Referring to Table 3 we have

\[ \theta_s = \frac{T_{1000}^{1000^{0.288}}}{\exp \frac{d\theta}{dT} TI_{1000}} \] (13)

Finally, by following this saturation adiabat to the surface pressure PD and to 1000 mb, we get the wet bulb temperature \( TW \) and the wet bulb potential temperature \( \theta W \) respectively

\[ TW = T_{SA}(\theta_s, PS) \] (14)

\[ \theta W = T_{SA}(\theta_s, 1000) \] (15)

5.6. The Pseudo Wet Bulb Temperature and Pseudo Wet Bulb Potential Temperature \( TPW \) and \( \theta PW \)

Refer to the wet bulb temperature and wet bulb potential temperature above.

5.7. The equivalent potential Temperature \( \theta_e \)

The equivalent potential temperature is computed from the same quantities used to compute the wet bulb temperature, i.e. the surface pressure, dew point temperature, and actual temperature. First compute the wet bulb temperature \( TW \). The equivalent potential temperature can then be computed by the same process used to determine the parameter \( \theta_s \) of a saturation adiabat through \( (TW,PS) \). Referring to Table 3, we have
\[ \theta_e = \frac{T_W^{\frac{1000}{P_S}^{0.288}}}{\exp\left(\frac{bW(T,W,P_S)}{TW}\right)} \]  

### 5.8. The Pseudo Equivalent Temperature TE

First the equivalent potential temperature \( \theta_e \) is computed. The pseudo equivalent temperature is then given by

\[ TE = \theta_e \left( \frac{P_S}{1000} \right)^{0.288} \]  

### 5.9. Thickness of a Layer Z

It is assumed that the temperature and dew point temperature are known at \( N \) distinct, decreasing pressures. Thicknesses are computed in meters from the surface. The trapezoidal rule is used to integrate

\[ Z = \frac{R}{0.98} \int_{\ln(P_T)}^{\ln(P_S)} \left[ T + \frac{0.6078 + W + T}{1000 + W} \right] d\ln P \]  

See Table 1 for a definition of \( W(T,P) \). Rewriting Eq. (18) and noticing that \( W \ll 1000 \) gives Eq. (19), which is used to compute \( Z \).

\[ Z = 29.2857 \left[ \frac{T_1 + T_2 + 6.078 \times 10^{-6} (W_1 T_1 + W_2 T_2)}{2} \ln \left( \frac{P_1}{P_2} \right) \right. \\
\left. + \frac{T_2 + T_3 + 6.078 \times 10^{-6} (W_2 T_2 + W_3 T_3)}{2} \ln \left( \frac{P_2}{P_3} \right) \right. \\
\left. + \frac{T_n + T_{n+1} + 6.078 \times 10^{-6} (W_n T_n + W_{n+1} T_{n+1})}{2} \ln \left( \frac{P_n}{P_{n+1}} \right) \right] \]  

### 5.10. The Lifting Condensation Level LCL

The lifting condensation level is computed in the same manner that PI was computed for the wet bulb temperature, using Eqs. (8), (9), (10), (11), and (12). \( (T_I, P_I) \) locate the LCL.

### 5.11. The Convective Condensation Level CCL

It is assumed that the temperature and dew point temperature are known at \( N \) distinct, decreasing pressures. The pressure at the top of the of the mixing ratio \( P_M \) must be greater than \( P_n \), the last pressure level. Since \( P_M \) is bounded by \( P_1 \) and \( P_n \), there is a \( K \) such that

\[ P_k > P_M \geq P_{k+1} \]  

First the mean mixing ratio \( W \) in the \( P_1 \)-PM layer is computed:

\[ W = \frac{\sum_{i=1}^{k-1} [W(T_i, P_i) + W(T_{i+1}, P_{i+1})] \ln(P_i) - \ln(P_{i+1})}{2(\ln(P_1) - \ln(P_{k+1}))} \]
\[ + \frac{[W(T_k, P_k) + W(T_m, P_M)] \ln(P_k) - \ln(P_M)}{2(\ln(P_1) - \ln(P_M))} \]  

The intersection of \( T_{MB}(W, P) \) and the curve defined by
\( T_S(P) = T_R - \frac{(T_{k+1} - T_k)\ln(P) - \ln(P_k)}{\ln(P_k) - \ln(P_{k+1})} \) (22)

(k is chosen such that \( P_k \geq P \geq P_{k+1} \)) defines the convective condensation level. This intersection can be found by first systematically comparing the difference between \( T_{MR}(W, P_i) \) and \( T_S(P_i) \) until the smallest is found such that

\[ T_{MR}(W, P_i) - T_S(P_i) < 0 \] (23)

and

\[ T_{MR}(W, P_{i+1}) - T_S(P_{i+1}) < 0 \] (24)

A bisection method is used to determine PC, the pressure at the CCL. An initial guess \( PC_1 \) is made, tested to see if \( T_{MR}(W, PC_1) \) equals \( T_S(PC_1) \), and if not, corrected.

\[ PC_1 = 0.5(P_i + P_{i-1}) \] (25)

\[ PC_j = PC_{j-1} + P^*_j \text{ (corrector)} \] (26)

\[ P^*_k = \frac{P_{i+1} + P_{i+1}}{2^k} \text{SIGN}(T_{MR}(W, PC_k) - T_S(P_k)) \] (27)

Ten corrections are made.

**5.12. The Convective Temperature CT**

First, the pressure \( PC \) at the convective condensation level is computed. The temperature at the CCL, \( TC \), is computed from \( PC \) and \( \bar{W} \):

\[ TC = T_{MR}(\bar{W}, PC) \] (28)

A dry adiabat is determined:

\[ \theta = TC \left( \frac{1000}{PC} \right)^{288} \] (29)

Finally, the convective temperature \( CT \) is computed from \( \theta \) and the surface pressure \( PS \):

\[ CT = \theta \left( \frac{PS}{1000} \right)^{288} \] (30)

**6. Applications**

The algorithms are useful for data reduction purposes. The memory and speed requirements are not excessive and most computations can be carried out successfully on a programmable calculator. In addition to data analysis, the algorithms are useful for generating backgrounds for the presentation of data. An example of a computer generated background and plotted sounding is given in Fig. 1.

Computation of selected meteorological quantities from the plotted sounding in Fig. 1 is presented in Table 4. A table of CCL temperatures, pressures and heights was computed using an arbitrary decrement of -25 mb for the pressure at the top of the mixing layer.
<table>
<thead>
<tr>
<th>Table 4: An Example of a Vertical Sounding</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pressure</strong></td>
</tr>
<tr>
<td><strong>Temperature</strong></td>
</tr>
<tr>
<td><strong>Dew Point</strong></td>
</tr>
<tr>
<td><strong>Height</strong></td>
</tr>
<tr>
<td><strong>Potential Temp</strong></td>
</tr>
<tr>
<td><strong>Relative Humidity</strong></td>
</tr>
<tr>
<td><strong>Mixing Ratio</strong></td>
</tr>
<tr>
<td><strong>Saturation VP</strong></td>
</tr>
<tr>
<td><strong>Vapour Pressure</strong></td>
</tr>
<tr>
<td><strong>Wet Bulb Temp</strong></td>
</tr>
<tr>
<td><strong>Potential W Bulb T</strong></td>
</tr>
<tr>
<td><strong>Equiv Pot Temp</strong></td>
</tr>
</tbody>
</table>

Lifting Condensation Level (LCL): Temperature 17.9, Pressure 983, Height 260

<table>
<thead>
<tr>
<th>Mixing Layer</th>
<th>Convective Condensation Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>Height</td>
</tr>
<tr>
<td>988</td>
<td>215</td>
</tr>
<tr>
<td>963</td>
<td>437</td>
</tr>
<tr>
<td>938</td>
<td>662</td>
</tr>
<tr>
<td>913</td>
<td>891</td>
</tr>
<tr>
<td>888</td>
<td>1121</td>
</tr>
<tr>
<td>863</td>
<td>1359</td>
</tr>
<tr>
<td>838</td>
<td>1602</td>
</tr>
<tr>
<td>813</td>
<td>1847</td>
</tr>
<tr>
<td>788</td>
<td>2099</td>
</tr>
<tr>
<td>763</td>
<td>2358</td>
</tr>
</tbody>
</table>

Units: temperature °C, pressure millibar, mixing ratio g/kg, height metres

Note: values have been corrected from the original document.

7. **Acknowledgements**

Many individuals and groups contributed stimulating discussion and valuable time in assisting the author on this study and it is impossible to name them all. Deep appreciation is extended to the National Center for Atmospheric Research, which is sponsored by the National Science Foundation, for computer time used in this research. Mr. Walter S. Nordquist, who provided the impetus for this study, as well as many suggestions along the way and a critical reading of the manuscript, deserves much of the credit for this work. Finally, Mr. Alex Blomerth provided much administrative assistance, without which this study would not have been possible.

8. **Addenda by Harold Reynolds**

8.1. **Corrections to Original Manuscript**

The original definition of $\theta_e$, Equation (16), does not agree with the standard definition,

$$\theta_e = \theta \exp \left( \frac{LW(TD,P)}{c_p T_{SAT}} \right)$$  

(34)
where $L$ is the latent head of condensation of water, $c_p$ is the specific heat of air at constant pressure, and $T_{SAT}$ is the temperature at which the air parcel becomes saturated when lifted adiabatically. This temperature can be readily found using the Lifting Condensation Level subroutine to find the pressure of the LCL, followed by finding the temperature on the appropriate dry adiabat at that pressure.

Table 4 of the original manuscript has been revised to reflect the values I found when running the thermodynamic subroutines on the sample sounding.

### 8.2. Thermodynamic Functions Visual Basic Code

Listed below are the various thermodynamic functions discussed in the paper, translated into Visual Basic from the original Fortran. Like with any code, you must check and verify the correctness of the code yourself before using it for any project. I was able to reproduce the results from the first table in Table 4, which is a hopeful sign.

```visualbasic
Option Explicit

' Thermodynamic chart software package.
' Developed by G. S. Stipanuk, Atmospheric Sciences Laboratory, White Sands
' Missile Range, New Mexico, 88002. Presented in
' "Algorithms for Generating
' a SKEW-T, log P diagram and computing selected meteorological quantities",
' U.S. Government publication ECOM-5515, published October, 1973, and available
' on microfiche from NTIS.

' The following subroutines approximate a thermodynamic chart.
' T is the temperature in Kelvin. Scalar in all functions except Z and CCL.
' TD is the dew point temperature. Ditto.
' P is the pressure in millibars. Ditto.
' TDS, TS, and PS are TD, T, and P at the surface.
' WBAR is the mean mixing ratio.
' O is really a theta.
' Soundings must be ordered by decreasing pressure.

' Harold's note: function names are capitalized, variables are not.
' LEGAL STUFF: To the best of my knowledge, these routines produce the correct
' results. However, YOU MUST VERIFY THEIR
' CORRECTNESS BEFORE USING THESE
' FUNCTIONS FOR ANY PROJECT! IF SOMETHING IS WRONG,
' I WOULD VERY MUCH LIKE
' TO CORRECT IT! YOU HAVE BEEN WARNED!

' Thermodynamic chart software package.
' Developed by G. S. Stipanuk, Atmospheric Sciences Laboratory, White Sands
' Missile Range, New Mexico, 88002. Presented in
' "Algorithms for Generating
' a SKEW-T, log P diagram and computing selected meteorological quantities",
' U.S. Government publication ECOM-5515, published October, 1973, and available
' on microfiche from NTIS.

Function ALC(x As Double, y As Double) As Double
' Computes the pressure at the lifting condensation level.
' x, y in K, ps, alcl in mb. ABS = absolute value.
Dim aw As Double, ao As Double, pi As Double, x As Double
Dim i As Integer
aw = W(x, y)
ao = O(x, y)
pi = y
For i = 1 To 10
x = 0.02 * (TMR(aw, pi) - TDA(ao, pi))
If Abs(x) < 0.01 Then Exit For
pi = pi * 2 ^ x
Next i
ALCL = pi
End Function

Function CCL(pm As Double, p() As Double, t() As Double, td() As Double, wbar As Double, N As Integer) As Double
' Computes pressure at convective condensation level.
' N is the number of levels in the sounding. K is the last level below pm.
' Pm is pressure at top of mixing layer.
' CCL and p in mb, t in Kelvin, war in g vapour/kg dry air.
Dim t() As Double, td() As Double, p() As Double, wbar As Double, N As Integer
Dim t(0) As Double, td(0) As Double, p(0) As Double, wbar As Double
Dim k As Integer, j As Integer, i As Integer, L As Integer
Dim FoundIt As Boolean
wbar = 0
k = 1
Do While p(k) < pm
k = k + 1
Loop
k = k - 1
j = k - 1
If j >= 1 Then
' Compute the average mixing ratio. Log is natural logarithm
For i = 1 To j
L = i + 1
wbar = (W(td(i), p(i)) + W(td(L), p(L))) * Log(p(i) / p(L)) + wbar
Next i
wbar = wbar / (2 * Log(p(1) / pm))
End If

L = k + 1
tq = td(k) + (td(L) - td(k)) * Log(pm / p(k)) / Log(p(L) / p(k))
If Abs(tq - pm) > 0.01 Then Exit Function
wbar = wbar + (W(td(k), p(k)) + W(tq, pm)) * Log(p(k) / pm)
End If

' Find the level at which tmr = ts changes sign. TS is sounding temp.
```
Computes the dry adiabat through \((t, p)\) in Kelvin.

End Function

Function ESAT\(t\) As Double, t in K. Log to base 10 is needed for a separate function for it here.

\[\text{ESAT} = 10^{(a0 + a1 + a2)} \times \text{Log}(p/L) / pc + 273.16\]

For \(i = 1\) To 10
\[\text{del} = \text{del} / 2\]
\[x = \text{TMR}(wbar, pc) - t(L) = a \times \text{Log}(p(L) / pc) + 273.16\]
\[\text{CCL} = \text{pc}\]
End Function

Function CT\(wbar\) As Double, pc As Double, ps As Double)
"Computes the convective temperature."
\[\text{Wbar in g/kg, pc, ps in mb.}\]
Dim tc As Double, ao As Double

\[tc = \text{TMR}(wbar, pc) + 273.16\]
\[ao = O(tc, pc)\]
CT = TDA(ao, ps)
End Function

Function ESAT\(t\) As Double, ts As Double, ps As Double; FO = TSAT(atw, ps) - 273.16
End Function

Function CT\(wbar\) As Double, pc As Double, ps As Double)
"Computes the potential equivalent / pseudo-equivalent temperature."
\[\text{t, ts, oe in K, ps in mb.}\]
\[\text{NOTE: The commented formula is the one from the paper and gives wrong results!}\]
\[\text{I have used instead the formula from Holton, p. 331, with \(q\) as the mixing ratio of the parcel and \(T\) as the saturation temperature (temperature at the LCL).}\]
Dim alift As Double, olift As Double, tlift As Double

\[\text{alift} = \text{ATCL}(tds, ts, ps)\]
\[\text{olift} = O(ts, ps)\]

\[\text{tlift} = \text{TDA}(olift, alift) + 273.16\]
\[\text{OE} = O(ts, ps) \times \exp(2.6518986 \times W(tds, ps) / tlift) - 273.16\]
End Function

Function OS\(t\) As Double, p As Double)
"Computes saturation adiabat through \((t, p)\)
\[\text{OS, t in K, p in millibars (mb)}\]
\[\text{OE} = O(ts, ps) \times \exp(-2.6518986 \times W(t, p) / t)\]
End Function

Function OW\(tds\) As Double, ts As Double, ps As Double)
"Computes potential wet bulb temperature."
\[\text{t, ts and OW in K, ps in mb.}\]
Dim atw As Double, aos As Double

\[\text{atw} = \text{TW}(tds, ts, ps) + 273.16\]
\[\text{aos} = \text{OS(atw, ps)}\]
\[\text{OW} = \text{TSA(aos, 1000)}\]
End Function

Function TDA\(o\) As Double, p As Double)
"Computes temperature on a dry adiabat \((\theta)\) at temperature \((\theta)\) in K, p in mb."
Dim aoe As Double

\[\text{aoe} = \text{OE}(atw, 1000) - 273.16\]
End Function

Function FO\(t\) As Double, p As Double)
"Computes relative humidity. \(FR\) in percent, \(t, t\) in Kelvin."
FR = ESAT\(t\) / ESAT\(t\) * 100
End Function

Function OE\(t\) As Double, p As Double)
"Computes the dry adiabat through \((t, p)\)
\[\text{O and T in K, p in mb.}\]

O = t \times (1000 / p) ^ 0.288
End Function
'Computes temperature on mixing ratio w at pressure p.
'TMR in C, w in g/kg dry air, p in millibars.
Dim x As Double
    x = Log10(W * p / (622 + W))
    TMR = 10 ^ (0.0498646455 * x + 2.4082965) -
        (10 ^ (0.0915 * x - 1.2035) ^ 2)
End Function

Function TSA(OS As Double, p As Double) As Double
'Computes temperature on saturated adiabat os at
'pressure p.
'SIGN(a,b) replaces the algebraic sign of a with
'that of b.
Dim a As Double, tq As Double, d As Double, x As Double
Dim i As Integer
    a = OS
    tq = 253.16
    d = 120
    For i = 1 To 12
        d = d / 2
        x = a * Exp(-2.6518986 * W(tq, p) / tq -
            2.6518986 * W(253.16, p) / 253.16) -
            253.16 * (1000 / p) ^ 0.288
        If Abs(x) <= 0.0000001 Then Exit For
        tq = tq + SIGN(d, x)
    Next i
    TSA = tq - 273.16
End Function

Function TW(tds As Double, ts As Double, ps As Double) As Double
'Computes wet bulb temperature.
'tds, ts and TW in K, ps in mb.
Dim i As Integer
Dim aw As Double, ao As Double, pi As Double, x As Double
Dim ti As Double, aos As Double
    aw = W(tds, ps)
    ao = O(ts, ps)
    pi = ps
    For i = 1 To 10
        x = 0.02 * (TMR(aw, pi) - TDA(ao, pi))
        If Abs(x) <= 0.01 Then Exit For
        pi = pi + 0.02 * x
    Next i
    ti = TDA(ao, pi) + 273.16
End Function

'The intersection has been found, now find a
'saturated adiabat through it.
    aos = OS(ti, pi)
    TW = TSA(aos, ps)
End Function

'--------------------------------------------------------
Function W(t As Double, p As Double) As Double
'Computes the mixing ratio line through (t,p).
'T is in K, p in mb, W in g water /kg dry air.
Dim x As Double
    x = ESAT(t)
    W = 622 * x / (p - x)
End Function

'--------------------------------------------------------
Function Z(pt As Double, p() As Double, t() As Double, td() As Double, N As Integer) As Double
'Computes thickness in metres from p(1) to pt.
Dim i As Integer, j As Integer
Dim a1 As Double, a2 As Double, Z1 As Double
Z1 = 0
    For i = 1 To N
        j = i + 1
        If pt >= p(j) Then Exit For
        a1 = t(j) * (1 + 0.0006078 * W(td(j), p(j)) -
            0.0006078 * W(td(i), p(i)))
        a2 = t(i) * (1 + 0.0006078 * W(td(i), p(i)) -
            0.0006078 * W(td(i), p(i)))
        Z1 = Z1 + 14.64285 * (a1 + a2) * Log(p(i) / p(j))
    Next i
    a1 = t(j) * (1 + 0.0006078 * W(td(j), p(j)) -
        0.0006078 * W(td(i), p(i)))
    a2 = t(i) * (1 + 0.0006078 * W(td(i), p(i)) -
        0.0006078 * W(td(i), p(i)))
    Z = Z1 + 14.64285 * (a1 + a2) * Log(p(i) / pt)
End Function

'--------------------------------------------------------
Function Log10(ByVal x As Double) As Double
'Computes logarithm to base 10
    Log10 = Log(x) / Log(10)
End Function

'--------------------------------------------------------
Function SIGN(x As Double, y As Double) As Double
'Replaces the sign of x with that of y. This is
'used to mimic the intrinsic
'function SIGN in Fortran.
    If y < 0 Then
        SIGN = -Abs(x)
    Else
        SIGN = Abs(x)
    End If
End Function
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